

Orthogonal Steiner systems

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Abstract

The research on orthogonal Steiner systems $S(t, k, v)$ was initiated in 1968. For $(t, k) \in \{(2, 3), (3, 4)\}$, this corresponds to orthogonal Steiner triple systems (STSs) and Steiner quadruple systems (SQSs), respectively. The existence problem of a pair of orthogonal STSs or SQSs has been settled completely thirty years ago. However, for Steiner systems with $t \geq 3$ and $k \geq 5$, only two small examples of orthogonal pairs were known to exist before this work. An infinite family of orthogonal Steiner systems $S(3, 5, v)$ is constructed here. In particular, the existence of a pair of orthogonal Steiner systems $S(3, 5, 4^m + 1)$ is established for any even $m \geq 2$; in parallel a pair of orthogonal G-designs $G(\frac{4^m+1}{5}, 5, 5, 3)$ is displayed for any odd $m \geq 3$. The construction is based on the Steiner systems admitting 3-transitive automorphism groups supported by elementary symmetric polynomials. What's more, 50 mutually orthogonal Steiner systems $S(5, 8, 24)$ are shown to exist. (Join work with Qianqian Yan.)

Keywords: Steiner system, G-design, symmetric polynomial, disjoint, orthogonal