Transferable domination number of graphs

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Abstract

Let G be a connected graph, and let $\mathcal{D}(G)$ be the set of all dominating (multi)sets for G. For D_1 and D_2 in $\mathcal{D}(G)$, we say that D_1 is single-step transferable to D_2 if there exist $u \in D_1$ and $v \in D_2$, such that $uv \in E(G)$ and $D_1 - \{u\} = D_2 - \{v\}$. We write $D_1 \stackrel{*}{\longrightarrow} D_2$ if D_1 can be transferred to D_2 through a sequence of single-step transfers. We say that G is k-transferable if $D_1 \stackrel{*}{\longrightarrow} D_2$ for any D_1, $D_2 \in \mathcal{D}(G)$ with $|D_1| = |D_2| = k$. The transferable domination number of G is the smallest integer k to guarantee that G is l-transferable for all $l \geq k$. We study the transferable domination number of graphs in this paper. We give upper bounds for the transferable domination number of general graphs and bipartite graphs, and give a lower bound for the transferable domination number of grids. We also determine the transferable domination number of $P_m \times P_n$ for the cases that $m = 2, 3$, or $mn \equiv 0 \pmod 6$. Beside these, we give an example to show that the gap between the transferable domination number of a graph G and the smallest number k so that G is k-transferable can be arbitrarily large.

Keywords: dominating set, domination number, transferable domination number, grid.

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