

Some new results on spectral Turán-type problems

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Abstract

For a simple graph F , let $\text{Ex}(n, F)$ and $\text{Ex}_{\text{sp}}(n, F)$ denote set of graphs with the maximum number of edges and the set of graphs with the maximum spectral radius in an n -vertex graph without any copy of the graph F , respectively. The Turán graph $T_{n,r}$ is the complete r -partite graph on n vertices where its part sizes are as equal as possible. Cioabă, Desai and Tait [The spectral radius of graphs with no odd wheels, *European J. Combin.*, 99 (2022) 103420] posed the following conjecture: Let F be any graph such that the graphs in $\text{Ex}(n, F)$ are Turán graphs plus $O(1)$ edges. Then $\text{Ex}_{\text{sp}}(n, F) \subset \text{Ex}(n, F)$ for sufficiently large n . In this talk, we consider the graph F such that the graphs in $\text{Ex}(n, F)$ are obtained from $T_{n,r}$ by adding $O(1)$ edges, and prove that if G has the maximum spectral radius among all n -vertex graphs not containing F , then G is a member of $\text{Ex}(n, F)$ for n large enough. Thus Cioabă, Desai and Tait's conjecture is completely solved. We also give the spectral extremal graphs for (k, r) -fan and the unique spectral extremal graph for $kK_r + 1$.

Keywords: Spectral radius, Spectral extremal graph, Turán graph.